ANALYTIC EXPRESSIONS FOR THE LIGHT-SCATTERING CROSS SECTION AND ÅNGSTRÖM EXPONENT OF AN AEROSOL

Ernie R. Lewis

elewis@bnl.gov

Brookhaven National Laboratory, Upton, NY 11933

BACKGROUND

Light scattering by an aerosol consisting of spherical particles with the same composition and with size distribution of number concentration $dN(r)/dr$ is characterized by the light-scattering coefficient σ_{sp}, defined by

$$\sigma_{sp} = \int \pi r^2 Q_{sp}(r/\lambda, m)\,\frac{dN(r)}{dr}\,dr.$$

Q_{sp} is the scattering efficiency of a particle, which depends on the ratio of the radius r to the wavelength λ and on the refractive index m, assumed to be real (thus no absorption).

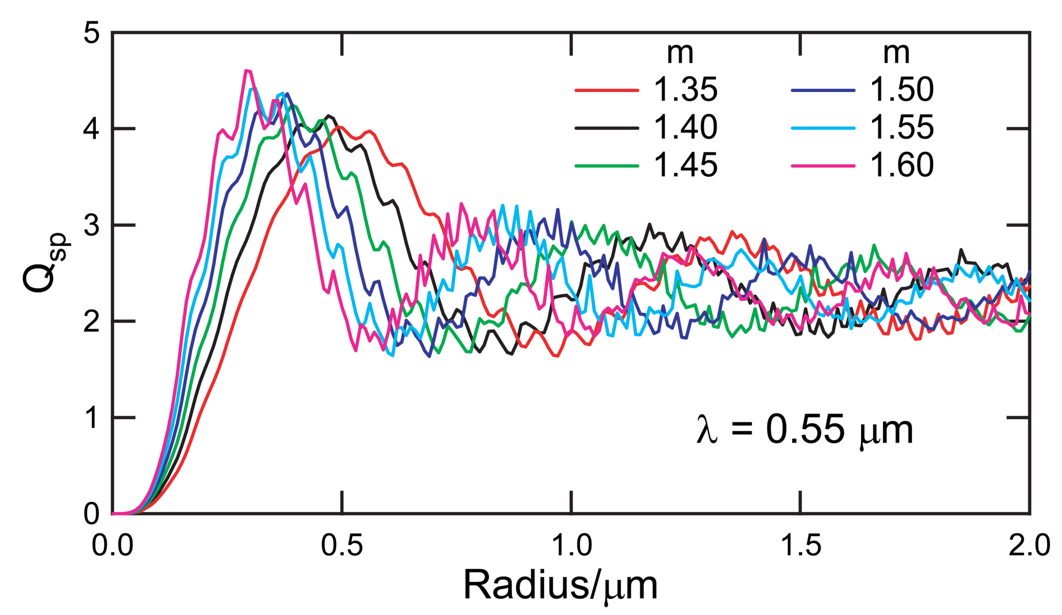
- Q_{sp} varies as $(r/\lambda)^4$ for particles with $r \ll \lambda$
- Q_{sp} approaches the value 2 for particles with $r \gg \lambda$

The wavelength dependence of σ_{sp} is characterized by the Ångström exponent $å$, defined by

$$å \equiv \frac{-\partial \ln \sigma_{sp}}{\partial \ln \lambda} = \frac{-\lambda}{\sigma_{sp}} \int \pi r^2 \frac{\partial Q_{sp}(r/\lambda, m)}{\partial \lambda}\,\frac{dN(r)}{dr}\,dr.$$

- $å = 4$ when most scattering is from particles with $r \ll \lambda$
- $å = 0$ when most scattering is from particles with $r \gg \lambda$

Q_{sp} thus determines many light-scattering properties of interest.



Q_{sp} is determined by Mie theory. It is a quite complicated function of r/λ and m, with many "wiggles" on many scales, and its computation, though straightforward, is time intensive.

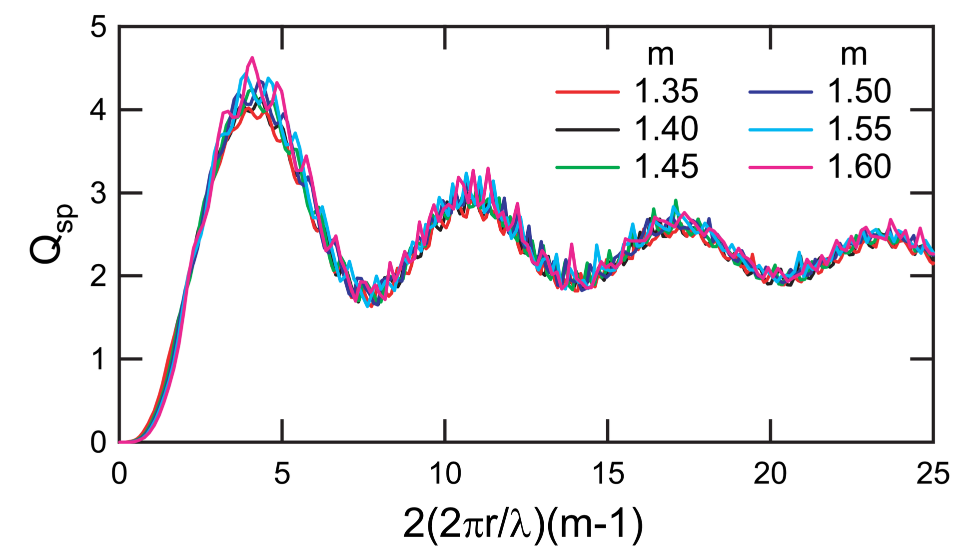
A simple expression for Q_{sp} would simplify calculation of many aerosol optical properties and would provide insight into the dependence of these properties on wavelength, index of refraction, RH, and the characteristic size of the aerosol.

SIMPLIFICATION OF Q_{sp}

Much of the variability in Q_{sp} can be eliminated by plotting it against the quantity

$$z \equiv 2\frac{2\pi r}{\lambda}(m-1)$$

- for $\lambda = 0.55$ µm and $m = 1.40$, $z \approx 9(r/\mu m)$
- small wiggles don't matter, as they are smoothed out by the spread in the size distribution



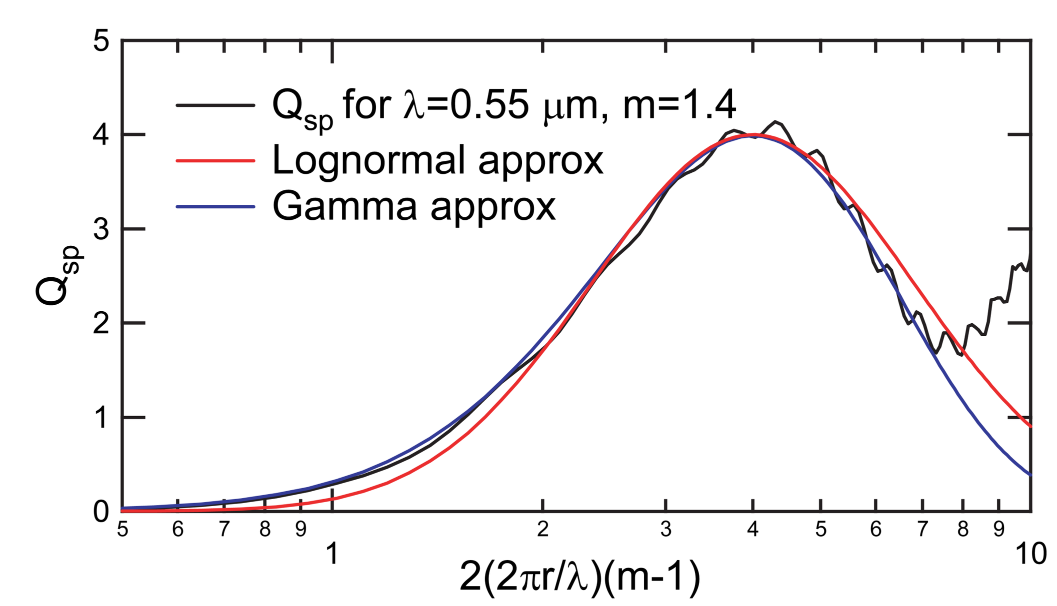
APPROXIMATION OF Q_{sp}

In many situations the majority of the scattering is from the range $1 < z < 8$ (~0.1 < r/µm < ~0.7). Over much of this range, Q_{sp} can be accurately approximated by a lognormal with $z = 4$ with geometric standard deviation 1.7:

$$Q_{LN} = 4\exp\left(\frac{-[\ln(z/4)]^2}{2[\ln 1.7]^2}\right)$$

or by a gamma function with effective variance 1/7:

$$Q_{\Gamma} = 0.85 z^4 e^{-z}.$$



Both provide excellent approximations for $\sim 1 < z < \sim 8$.

CALCULATION OF σ_{sp}

For a lognormal size distribution with

$$\frac{dN}{dr} = \frac{N_{tot}}{(2\pi)^{1/2}\, r \ln\sigma} \exp\left\{\frac{-1}{2}\left[\frac{\ln(r/r_0)}{\ln\sigma}\right]^2\right\}$$

the lognormal approximation for Q_{sp} yields for the light-scattering cross section

$$\sigma_{sp} = \frac{4\pi N_{tot}}{a^{1/2}\ln\sigma}\left[\frac{\lambda}{4\pi(m-1)}\right]^2 \exp\left(\frac{b^2 - ac}{2a}\right)$$

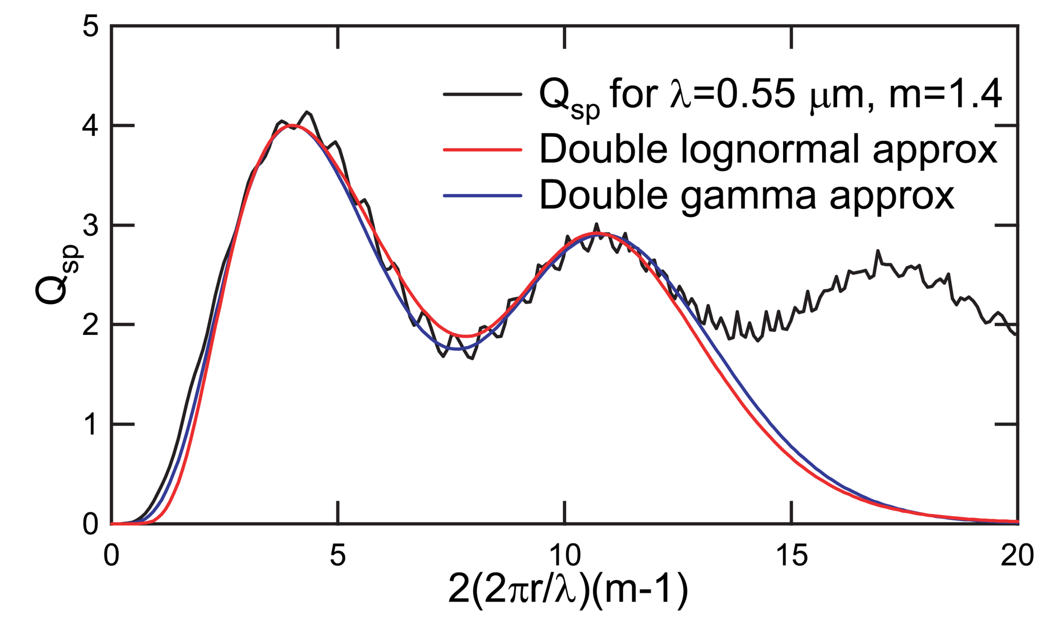
where

$$a = \frac{1}{\ln^2 1.7} + \frac{1}{\ln^2\sigma}$$

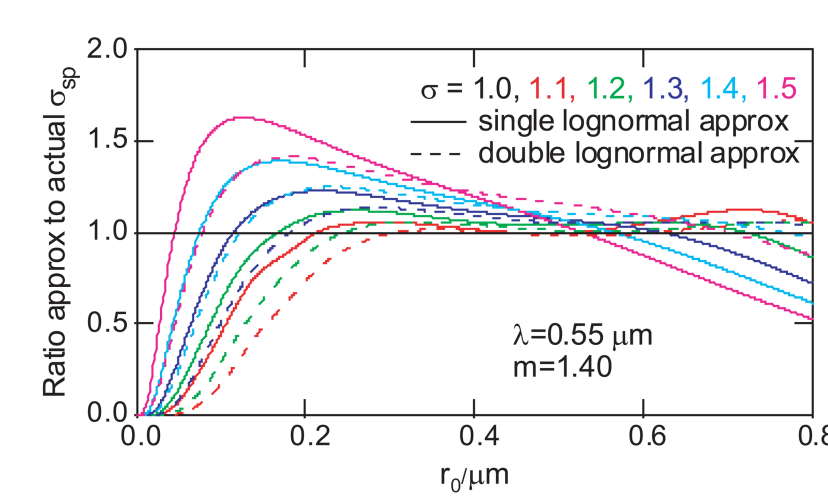
$$b = \frac{\ln 4}{\ln^2 1.7} + \frac{\ln[4\pi r_0(m-1)] - \ln\lambda}{\ln^2\sigma} + 2$$

$$c = \frac{\ln^2 4}{\ln^2 1.7} + \frac{\{\ln[4\pi r_0(m-1)] - \ln\lambda\}^2}{\ln^2\sigma}$$

This can be extended by approximating Q_{sp} as a sum of several lognormals or gamma functions.



HOW GOOD ARE THE APPROXIMATIONS?



Accuracy is within ~20% over a wide range of r_0, σ, but generally decreases with increasing σ as more scattering is outside the range for which the approximation is accurate.

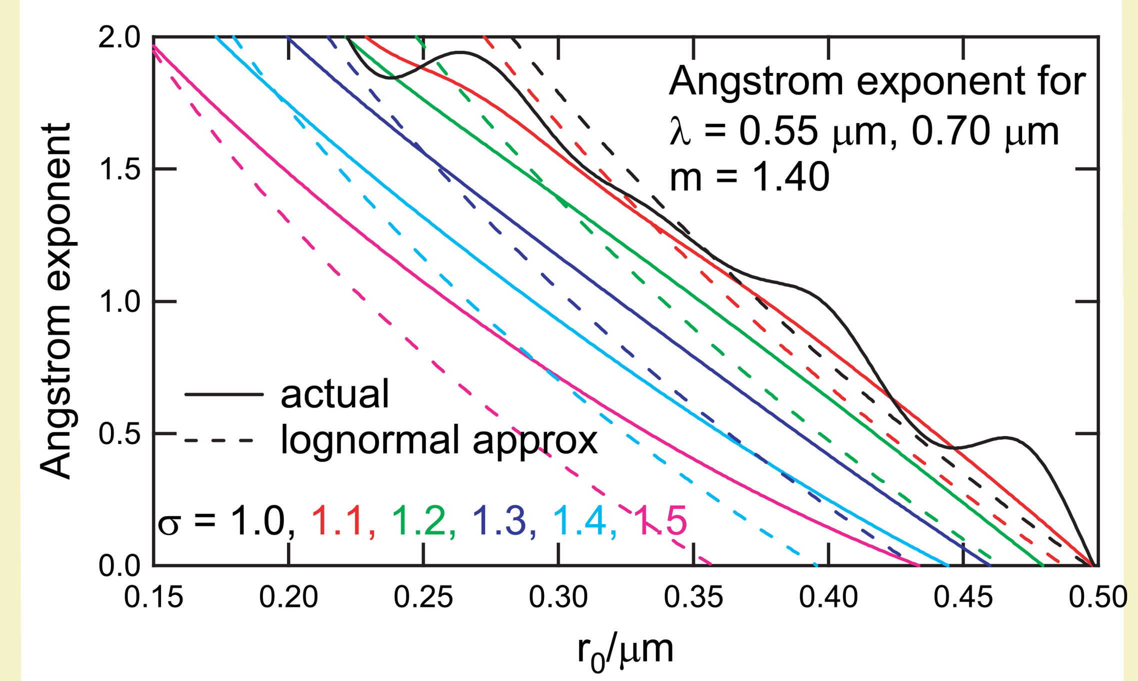
CALCULATION OF $å$

The Ångström exponent $å$ can be calculated for a lognormal size distribution using the lognormal approximation for Q_{sp} as

$$å = \frac{\ln 4 - \ln\left\{\dfrac{4\pi\left[r_0 \exp\left(2\ln^2\sigma\right)\right](m-1)}{\lambda}\right\}}{\ln^2 1.7 + \ln^2\sigma}$$

- $å$ depends on the quantities $r_0 \exp(2\ln^2\sigma)$ and $\ln^2\sigma$
- $å$ increases with increasing λ
- The dependence of $å$ on RH is not simple, as r_0 increases but m decreases with increasing RH.

HOW GOOD IS THIS APPROXIMATION?



- Most measurements of $å$ yield values between 0 and 2.
- The lognormal approximation underestimates $å$, implying that the dependence of σ_{sp} on λ is greater than estimated.
- The two-lognormal approximation is more accurate.

CONCLUSIONS

- Q_{sp} depends mostly on $z \equiv 4\pi(r/\lambda)\cdot(m-1)$.
- Q_{sp} can be accurately approximated by lognormals or gamma functions.
- These approximations yield analytic expressions for σ_{sp} and its dependence on aerosol properties such as r_0 and σ and on m, RH, and λ (yielding $å$).